Credit Portfolio Modelling, Marginal Risk Contributions, and Granularity Adjustment

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Abstract

This paper first provides a simple but very general framework for credit portfolio modelling which is based on the distinction between systematic and unsystematic risk. Unsystematic or borrower-specific risk vanishes through diversification in a very large, infinitely fine-grained portfolio. The framework contains typical models like CreditRisk+ and CreditMetrics as special cases. An analysis of marginal risk contributions is then done which also includes a theoretical formula for the granularity adjustment in a "lumpy" credit portfolio.

JEL classification: D 81, G 21, G 28

1. Introduction

The standard tool for credit portfolio management is today Value at Risk (VaR), which is defined as the quantile of the profit and loss distribution for a given confidence level: For a confidence level of e.g. p=99%, one is 99% certain that at the end of the plannig horizon there will be no greater loss than just the VaR. If VaR is completely covered by equity capital, the confidence level is the minimum probability that insolvency will not occur[1]. In practice, VaR for a credit loan portfolio is calculated with models like CreditRisk+ (1997) from Credit Suisse First Boston or CreditMetrics (1997) from JP Morgan. Recently, the Basel Committee on Banking Supervision (2001) has also adopted VaR in the proposals for a new capital accord.

In the past, some researchers have observed the given similarities between different credit risk models. Koyluoglu and Hickman (1998) and Finger (1999) have pointed to the fact that for given realizations of the background factors or systematic risk factors, defaults and rating changes are generally assumed to be stochastically independent. Similar, Gordy (2000) has shown that a restricted two-state version of CreditMetrics, which differentiates only between default and non-default, can be mapped into the CreditRisk+ framework and vice versa. In this paper, I begin with a simple but very general framework for credit portfolio modelling which contains models like CreditRisk+ or the unrestricted multi-state version of CreditMetrics as special cases. In this framework, the value of each loan at the end of the planning horizon is a function of some systematic risk factors common to all borrowers and an additional specific or unsystematic risk factor. As a consequence of the law of large numbers, unsystematic risk vanishes through diversification in a very large, infinitely fine-grained portfolio.

An important question is how much additional equity capital is required if a single loan is added to the credit portfolio. In order to answer this question, the derivative of the VaR must be calculated. It can be shown mathematically that the derivative is given by the conditional mean of the marginal loan, on condition that the value of the credit portfolio and VaR are exactly identical. If this general result is applied to a simple one-factor model, the model used by the Basel Committee can be obtained. Another result is a theoretical formula for the granularity adjustment, which is needed to cover the remaining unsystematic risk. Such a formula was recently presented by Wilde (2001). In this paper, a different derivation of that formula through a Taylor expansion will be given.

This paper is organized as follows. Section 2 introduces a general framework for credit portfolio modelling. Section 3 explains the role of diversification in that framework. In Section 4, a general formula for marginal risk contributions will be presented and applied to a simple one-factor model. This will be done by assuming an infinitely fine-grained credit portfolio. Subsequently, a granularity adjustment for "lumpy" credit portfolios is considered.

2. A general credit portfolio model

Consider a portfolio of n loans with exposure sizes $A_1,.., A_n$. As a percentage of the exposure size, the difference between the actual value of each loan and the value at the end of the planning horizon (usually one year) is described by a random loss variable L_i. Let $L_i=L_i(X,\varepsilon_i)$ be given as a function of some systematic risk factors $X = (X_1,...,X_k)$, which represent the state of the economy and are common to all borrowers, and a specific or unsystematic risk factor ε_i. Each ε_i is assumed to be stochastically independent from all other systematic and unsystematic risk factors.

Obviously, such a very general approach contains typical models like CreditMetrics or CreditRisk+ as special cases. CreditMetrics for example is a mark-to-market model in which the value of each loan is a function of the borrower`s credit rating. Note that in our model an upgrading would result in a gain in market value and consequently in a negative value of the loss variable L_i. CreditMetrics assumes that rating changes are driven by an underlying asset value process. The return r_i of the assets of borrower i is explained as a linear combination of systematic and unsystematic risk factors:

$$r_i = w_{i1} X_1 + ... + w_{ik} X_k + \hat{w}_i \varepsilon_i \qquad (1)$$

[1] For example, the usual goal of a AA rating for the bank requires a confidence level of 99,97% (planning horizon one year).

The realization of the asset return r_i then determines the rating of the borrower[2], and the respective rating defines the value of the loan at the end of the planning horizon.

CreditRisk+ differentiates only between default and non-default. Default probabilities $p_i=p_i(X)$ are volatile[3] and in general given as a linear combination of some gamma-distributed background factors $X = (X_1,...,X_k)$:[4]

$$p_i(X) = w_{i1}X_1 + ... + w_{ik}X_k \tag{2}$$

Obviously, the background factors in CreditRisk+ play the same role as the systematic risk factors in CreditMetrics. To see the similarities, assume that the background factors determine a certain threshold $T_i(X)$ so that borrower i defaults if the corresponding unsystematic risk factor ε_i fulfils $\varepsilon_i<T_i(X)$. The threshold $T_i(X)$ has to be chosen so that the probalitity for this is exactly $p_i(X)$. It follows that in both models the value of each loan at the end of the planning horizon is given as a function of some systematic risk factors and an additional unsystematic risk factor.

3. Diversification

As a percentage of total exposure, the random loss of the entire portfolio at the end of the risk horizon is

$$L_P = \frac{\sum_{i=1}^{n} A_i L_i}{\sum_{i=1}^{n} A_i} \tag{3}$$

Now assume that the realizations of the systematic risk factors $X = (X_1,...,X_k)$ occur before the realizations of the unsystematic risk factors ε_i. If the values of the systematic risk factors are taken as given, L_P is a sum of stochastically independent random varia-

[2] The rating would be AAA if $r_i \geq T_{AAA}$, AA if $T_{AAA} > r_i \geq T_{AA}$ and so on, where the thresholds T_i must be chosen so that migration probabilities are in accordance with the historical transition matrix. In addition, because systematic risk factors are common to all borrowers, the approach also takes the stochastic dependence of rating migrations into account.

[3] For given realizations of the default probabilities, default events are assumed to be stochastically independent.

[4] Gordy (2000) p. 122.

bles. Thus, the central limit theorem can be applied. Conditional on X, the portfolio loss variable L_P is asymptotically normal-distributed with mean

$$\mu(L_P | X) = \frac{\sum_{i=1}^{n} A_i \; \mu(L_i | X)}{\sum_{i=1}^{n} A_i} \tag{4}$$

and variance

$$\sigma^2 (L_P | X) = \frac{\sum_{i=1}^{n} A_i^2 \; \sigma^2 (L_i | X)}{(\sum_{i=1}^{n} A_i)^2} \tag{5}$$

It is easy to show that if $0 < A_{min} < A_i < A_{max}$ and $\sigma^2 (L_i | X) < \sigma^2_{max}$ for all i with finite boundaries A_{max} and σ^2_{max}, then $\sigma^2 (L_P | X) \to 0$ as $n \to \infty$. For n sufficiently large, the variance tends to zero and the probalitiy for an arbitrary small deviation of L_P from the conditional mean $\mu(L_P | X)$ gets arbitrary small. This is, of course, nothing else than an application of the law of large numbers.

On condition that the values of the systematic risk factors are given, L_P becomes non-stochastic in a very large, infinitely fine-grained portfolio. Borrower-specific or unsystematic risk can thus be eliminated through diversification. The only remaining risk is systematic risk, that is the risk that the actual values of the systematic risk factors $X = (X_1,...,X_k)$ result in a higher or lower value of the conditional mean $\mu(L_P | X)$.

4. Marginal risk contributions

4.1 A general result

In banking practice, the marginal risk contribution if a new loan is added to a portfolio is often assumed to be proportional to the marginal standard deviation. From a theoretical perspective, this is obviously wrong because credit risk is by nature highly skewed and fat tailed. The standard deviation is therefore not an appropriate measure for credit risk. So what is needed is a general formula for marginal risk contributions which does not rely on specific assumptions about the loss distribution.

In order to formulate, first without any reference to the previous stated framework, such a general result, suppose that the value of the actual portfolio is given by a random variable *Y* and that a fraction *t* of another random variable *Z* is added to that portfolio. Then, the condition

$$Prob(Y + tZ > VaR(Y + tZ)) = \alpha = const. \tag{6}$$

that the actual realization of *Y+tZ* exceeds *VaR(Y+tZ)* only with a constant probability α implicitly defines *VaR(Y+tZ)* as a function of *t*. In appendix A, the first and second deriviatives of *VaR(Y+tZ)* with respect to *t* are calculated[5]. The only assumptions made is that the random variables *Y* and *Z* have a joint probability density function and that first and second moments exists. The first derivative is simply the conditional mean:

$$\frac{\partial VaR(Y + tZ)}{\partial t}\Big|_{t=0} = \mu(Z \mid Y = VaR(Y)) \tag{7}$$

Intuitively, this result can be interpreted as follows: If *Y>VaR(Y)* (the bank is already bankrupt) or *Y<VaR(Y)* (there is a remainig equity buffer) and for a sufficiently low value of *t*, adding a very small sufficiently small fraction *tZ* would not change the outcome. Therefore, the marginal capital requirement for an additional risk is the average value for all critical cases with *Y=VaR(Y)*.

As a special case, assume that *Y* and *Z* are bivariate normal distributed, i.e. the case when the standard deviation is in fact the right risk measure. In this case, the usual formula for the linear regression applies, and the conditional mean is exactly equal to:

$$\frac{\partial VaR(Y + tZ)}{\partial t}\Big|_{t=0} = \mu(Z \mid Y = VaR(Y)) = \mu(Z) + \frac{cov(Y,Z)}{\sigma^2(Y)}(VaR(Y) - \mu(Y)) \tag{8}$$

[5] For similar results see also Gourieroux et al. (2000), Tasche (1999).

Here, $cov(Y,Z)/\sigma^2(Y)$ is the usual beta-factor known from the classical CAPM. As VaR is commonly considered as the sum of expected and so-called unexpected loss, the formula states that marginal VaR is given by expected loss of the marginal loan plus beta-factor times unexpected loss of the portfolio. Of course, as already mentioned, the underlying assumption of a normal distribution is problematic when applied to the loss distribution of a credit loan portfolio.

4.2 One-factor model

Above, a general formula has been derived which states that marginal VaR is the conditional mean of the marginal risk, on condition that the value of the original portfolio exactly equals VaR. If applied to the credit risk framework developed earlier, the condition that the portfolio value equals VaR would impose a restriction on the choice of the risk factors. For a simple case, assume that

1) the value of each loan at the end of the planning horizon is an increasing function of only one systematic risk factor *X* (the loss variable L_i is then a decreasing function of *X*), i.e. *X* is a scalar

2) unsystematic risk is perfectly diversified away, i.e. $L_P = \mu(L_P | X)$

In this case, the only remainig risk is that the actual realization of *X* will be below the quantile x_α, with x_α implicitly defined by $Prob(X < x_\alpha) = \alpha$. The restriction imposed on the risk factor is simply $X = x_\alpha$. Therefore, as a percentage of total exposure, VaR of the whole credit portfolio is given by:

$$VaR(L_P) = \mu(L_P | X = x_\alpha) = \frac{\sum_{i=1}^{n} A_i \ \mu(L_i | X = x_\alpha)}{\sum_{i=1}^{n} A_i} \tag{9}$$

Marginal VaR for each Euro borrowed to borrower *i* is then given by the conditional mean of the individual loan $\mu(L_i | X = x_\alpha)$, with the condition that the systematic risk factor *X* equals the quantile x_α.

It follows that in such a one-factor model marginal risk contributions depend only on the characteristics of the individual loan, and not on the characteristics of the portfolio to which it is added. This is the reason why such a one-factor model has been adopted by the Basel Committee in the proposals for a new capital accord. If instead a multi-factor model had been used, the marginal risk contributions of each loan would also depend on how well the credit portfolio is diversified over the different sectors (countries or industries), with the state of each sector being represented by one of the systematic risk factors. It would be difficult for the regulator to obtain such detailed information about individual bank portfolios.

The model used by the Basel Committee is a simplified CreditMetrics model which differentiates only between default and non-default[6]. Default occurs if the asset return falls below a certain threshold D:

$$r_i = \sqrt{\rho}\ X + \sqrt{1-\rho}\ \varepsilon_i < D \qquad (10)$$

Here, ρ is the correlation coefficient of the asset returns and X, ε_i are independent standard normal distributed random variables with mean zero and variance one. Then, as a consequence of the choice of the coefficients, r_i is also standard normal distributed. The relationship between the default threshold D and the probability of default PD is $PD = N^{-1}(D)$, where N is the cumulative distribution function for a standard normal random variable. With default resulting in a loss given default LGD_i (as a percentage of the exposure A_i), marginal VaR is given as follows as the conditional mean, on condition that $X = x_\alpha = N^{-1}(\alpha)$:

$$\begin{aligned}\mu(L_i \mid X = x_\alpha) &= LGD_i\ Prob(\varepsilon_i < \frac{N^{-1}(PD) - \sqrt{\rho}\ x_\alpha}{\sqrt{1-\rho}}) \\ &= LGD_i\ N(\frac{N^{-1}(PD) - \sqrt{\rho}\ x_\alpha}{\sqrt{1-\rho}})\end{aligned} \qquad (11)$$

[6] The model is due to Vasicek (1997). See also Schonbucher (2001).

For example, in the consultative paper from January 2001, the Basel Committee has set $\rho = 0.2$ and $x_{0.5\%} = -2.57$ for a corporate loan portfolio (This will be probably not the parameter choice in the final accord). Formula (11) is then used to calculate the capital charge for a loan with probability of default PD .

4.3 Granularity adjustment in a one-factor model

Because no real-world portfolio can be infinitely fine-grained, a granularity adjustment has to be added to account for the remaining unsystematic risk, i.e. for large concentrations of risk in a "lumpy" portfolio. Such a granularity adjustment has also been proposed by the Basel Committee in the already mentioned consultative paper from January 2001. There, the calculation of the granularity adjustment is based on a theoretical result of Gordy (2001), who shows that the remaining unsystematic risk is inversely proportional to the effective number of loans. Gordy also estimates the proportional constant for typical loan portfolios numerically through Monte Carlo simulations. A theoretical formula for the granularity adjustment was recently given by Wilde (2001). Here I take a different approach which leads exactly to the same result as in Wilde (2001).

It has been shown above that VaR is given as the conditional mean, on condition that $X = x_\alpha$. The trick is then to develop a second-order Taylor expansion with respect to the error term $L_P - \mu(L_P|X)$. This results in (see appendix B):

$$VaR(L_P) \approx \mu(L_P|X = x_\alpha) - \frac{1}{2}\frac{\partial\sigma^2(L_P|X = x)/\partial x}{\partial\mu(L_P|X = x)/\partial x}\bigg|_{x = x_\alpha} - \frac{\sigma^2(L_P|X = x_\alpha)}{2}\frac{\partial \ln f_\mu(\mu)}{\partial\mu}\bigg|_{\mu = \mu(L_P|X = x_\alpha)} \quad (12)$$

where $f_\mu(\mu)$ denotes the probability density function of the conditional mean $\mu = \mu(L_P|X)$, which is a function of the systematic risk factor X .

The granularity adjustment consists of two terms: The sensitivity $(\partial\sigma^2 / \partial X)/(\partial\mu / \partial X) = \partial\sigma^2 / \partial\mu$ of the conditional variance with respect to the conditional mean and the conditional variance $\sigma^2(L_P|X)$ times the derivative of the logarithmic density $ln\, f_X$. The second term is positve if the density of the conditional mean $\mu = \mu(L_P|X)$ slopes downwards in the right tail, i.e. for very high average losses. This will usually be the case. Unclear is the sign of the first term. To get an intuition, note that the remaining unsystematic risk could also lift the value of the credit portfolio above the VaR-threshold if a violation of that threshold would otherwise occur. If $\partial\sigma^2 / \partial\mu$ is positive (the variance is an increasing function of average losses), the chance that the remaining unsystematic risk prevents a violation of the VaR-threshold is greater than the corresponding risk that a violation of the VaR-threshold is triggered only by unsystematic risk. As a consequence, it cannot be completely ruled out that the granularity adjustment might be negative, at least theoretically.

A simple example is a model with variable default probabiltity $p(X)=X$, and with $A_i=1$, $LGD_i=100\%$ for all i. Then $\sigma^2(L_i|X) = X(1-X)$ and equation (13) reduces to:

$$VaR(L_P) \approx x_\alpha - \frac{1-2x_\alpha}{2n} - \frac{x_\alpha(1-x_\alpha)}{2n}\frac{\partial\, ln\, f_X(x)}{\partial x}\bigg|_{x=x_\alpha} \qquad (13)$$

Here, the first part of the granularity adjustment will be in fact negative in most practical cases where the worst possible default probability $p(x_\alpha) = x_\alpha$ is lower than 50%. In addition, the granularity adjustment is inversely proportional to the number of loans n, which confirms the above mentioned result of Gordy (2001).

5. Conclusion

In recent years, practitioners have developed many different credit portfolio models. Here, a general framework for credit portfolio modelling has been developed which is based on the distinction between systematic and unsystematic risk. As a consequence of the law of large numbers, unsystematic risk can be completely diversified away in a very large, infinitely fine-grained portfolio. VaR and marginal risk contributions then depend only on systematic risk.

A simple case is a one-factor model where the systematic risk factor is a scalar. Then, if unsystematic risk is perfectly diversified away, the only remaining risk is that the realization of the systematik risk factor will be below the respective quantile. However, because no real-world credit portfolio is infinitely fine-grained, an additional granularity adjustment has to be added to account for large concentrations of risk in "lumpy" credit portfolios. As has been shown, the impact of the remainig unsystematic risk can be added incrementally rather than calculating both risks at once.

Mathematical appendix

A. First and second derivative of Value at Risk

Consider two random variables *Y* and *Z* with a joint probability density function *f(y,z)* and define *VaR=VaR(Y+tZ)* as a function of a real variable *t* by

$$Prob(Y + tZ > VaR) = \alpha = const.$$

Then:

$$\frac{\partial VaR}{\partial t} = \mu(Z \mid Y + tZ = VaR)$$

$$\frac{\partial^2 VaR}{\partial t^2} = -\left[\frac{\partial \sigma^2(Z \mid Y + tZ = s)}{\partial s} + \sigma^2(Z \mid Y + tZ = s)\frac{\partial \ln f_{Y+tZ}(s)}{\partial s}\right]_{s = VaR}$$

where $f_{Y+tZ}(s)$ denotes the probability density function of $Y + tZ$.

<u>Proof:</u>

Note first that the formula for the conditional density is:

$$f_Z(z \mid Y + tZ = VaR) = \frac{f(VaR - tz, z)}{f_{Y+tZ}(VaR)}$$

Then:

$$0 = \frac{\partial}{\partial t} Prob(Y + tZ > VaR)$$

$$= \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \int_{VaR - tz}^{+\infty} f(y, z)\, dy\, dz$$

$$= \int_{-\infty}^{+\infty} \left(\frac{\partial VaR}{\partial t} - z\right) f(VaR - tz, z)\, dz$$

$$= \left(\frac{\partial VaR}{\partial t} - \mu(Z \mid Y + tZ = VaR)\right) f_{Y+tZ}(VaR)$$

Dividing by $f_{Y+tZ}(VaR)$ yields the result for the first derivative. The formula for the second derivate can be get as follows:

$$0 = \frac{\partial^2}{\partial t^2} Prob(Y + tZ > VaR)$$

$$= \frac{\partial^2}{\partial t^2} \int_{-\infty}^{+\infty} \int_{VaR - tz}^{+\infty} f(y, z)\, dy\, dz$$

$$= \frac{\partial}{\partial t} \int_{-\infty}^{+\infty} \left(\frac{\partial VaR}{\partial t} - z\right) f(VaR - tz, z)\, dz$$

$$= \int_{-\infty}^{+\infty} \frac{\partial^2 VaR}{\partial t^2} f(VaR - tz, z) + \left(\frac{\partial VaR}{\partial t} - z\right) \frac{\partial f(VaR - tz, z)}{\partial t}\, dz$$

$$= \int_{-\infty}^{+\infty} \frac{\partial^2 VaR}{\partial t^2} f(VaR - tz, z) + (\frac{\partial VaR}{\partial t} - z)^2 \left. \frac{\partial f(s - tz, z)}{\partial s} \right|_{s = VaR} dz$$

$$= \int_{-\infty}^{+\infty} \frac{\partial^2 VaR}{\partial t^2} f_Z(z | Y + tZ = VaR) f_{Y+tZ}(VaR)$$

$$+ (\frac{\partial VaR}{\partial t} - z)^2 \left. \frac{\partial (f_Z(z | Y + tZ = s) f_{Y+tZ}(s))}{\partial s} \right|_{s = VaR} dz$$

$$= \frac{\partial^2 VaR}{\partial t^2} f_{Y+tZ}(VaR)$$

$$+ \int_{-\infty}^{+\infty} (\mu(Z | Y + tZ = VaR) - z)^2 \left. \frac{\partial f_Z(z | Y + tZ = s)}{\partial s} \right|_{s = VaR} f_{Y+tZ}(VaR) dz$$

$$+ \int_{-\infty}^{+\infty} (\mu(Z | Y + tZ = VaR) - z)^2 f_Z(z | Y + tZ = VaR) \left. \frac{\partial f_{Y+tZ}(s)}{\partial s} \right|_{s = VaR} dz$$

$$= \left\{ \frac{\partial^2 VaR}{\partial t^2} + \left[\frac{\partial \sigma^2(Z | Y + tZ = s)}{\partial s} + \sigma^2(Z | Y + tZ = s) \frac{\partial \ln f_{Y+tZ}(s)}{\partial s} \right]_{s = VaR} \right\} f_{Y+tZ}(VaR)$$

q.e.d.

B. Granularity Adjustment

With appendix A and the parameter choice

$$Y = \mu(L_P | X) = \frac{\sum_{i=1}^{n} A_i \, \mu(L_i | X)}{\sum_{i=1}^{n} A_i}, \; t = \frac{1}{\sqrt{\sum_{i=1}^{n} A_i}}, \; Z = \frac{L_P - Y}{t} = \frac{\sum_{i=1}^{n} A_i (L_i - \mu(L_i | X))}{\sqrt{\sum_{i=1}^{n} A_i}}$$

which ensures, under the assumptions made in section 3, that the conditional variance of Z is finite, a Taylor expansion around t=0 directly leads to the following result:

$$VaR(L_P) = VaR(Y + tZ)$$

$$\approx VaR(Y) + t\frac{\partial VaR(Y+tZ)}{\partial t}\bigg|_{t=0} + \frac{t^2}{2}\frac{\partial^2 VaR(Y+tZ)}{\partial t^2}\bigg|_{t=0}$$

$$= VaR(Y) + \mu(tZ \mid Y = VaR(Y))$$

$$- \frac{1}{2}\left[\frac{\partial\sigma^2(tZ \mid Y = s)}{\partial s} + \sigma^2(tZ \mid Y = s)\frac{\partial \ln f_Y(s)}{\partial s}\right]_{s = VaR(Y)}$$

$$= \mu(L_P \mid X_\alpha) + 0$$

$$- \frac{1}{2}\frac{\partial\sigma^2(L_P \mid X = x)/\partial x}{\partial\mu(L_P \mid X = x)/\partial x}\bigg|_{x = x_\alpha} - \frac{\sigma^2(L_P \mid X = x_\alpha)}{2}\frac{\partial \ln f_\mu(\mu)}{\partial\mu}\bigg|_{\mu = \mu(L_P \mid X = x_\alpha)}$$

with $f_\mu(\mu) = \dfrac{f_X(x)}{\partial\mu(L_P \mid X = x)/\partial x}$.   q.e.d.

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